Kyiv School of Economics

Algebraic Geometry

2025/2026, Spring Trimester

Course faculty	Piotr Achinger e-mail: pachinger@kse.org.ua
Department	Department of Mathematics
Study program	Masters in Mathematics
ECTS credits	6 (180 hours)
Class hours	60 hours, 20 lectures and 10 practices (80 min each)
Course language	English
Course format	Offline, with remote participation capabilities

Overview

Prerequisites

This is an advanced (Master level) class in pure mathematics, with a fair amount of theory building. Prior experience with proof-based courses in math is necessary. That said, the class does not have heavy prerequisites and should be accessible to undergraduates interested in the topic.

Both undergraduates and PhD students are welcome to attend if they meet the prerequisites. If you are interested in taking the course but aren't sure if this is the right time, consult with your mentor or the lecturer.

The key prerequisites for the course are *MATH530 Sheaves and Cohomology* and *MATH550 Commutative Algebra*. Students with no prior knowledge of the basics of sheaf theory and commutative algebra are welcome to attend, but they will have some catching up to do.

Background and course rationale

Algebraic geometry is the study of algebraically defined geometric objects, such as algebraic varieties (roughly, zero sets of systems of polynomial equations in many variables, over an algebraically closed field) or schemes (spaces which locally "look like" the prime spectrum of a commutative ring. It has a close relationship with commutative algebra, differential geometry, complex analysis, number theory, and representation theory.

Course aims

The goal of the course is to introduce the audience to the basic objects of study in algebraic geometry: algebraic varieties and schemes, and (sheaves of) modules over them. Emphasis will be put on concrete objects such as projective varieties. In order to keep the course focused, we will work toward a single main goal, which will be the proof of the Weil conjecture (Riemann hypothesis) for curves over a finite field.

Learning outcomes

The students will have a working knowledge of the basic objects of algebraic geometry, such as projective schemes, blowing up, divisors, line bundles, coherent sheaves etc., allowing them to attempt research problems in the area.

Course Structure

Every week there will be a problem sheet (homework). It is very important to attempt homework assignments regularly, in order to proceed with the understanding of the material in this course and obtain feedback about one's understanding of the subject.

During **practical sessions**, the students solve problems related to that week's material.

At the end of the course there will be a **final exam** lasting for 3 hours.

Course duration. 13 weeks (Jan 12–Apr 12), 6 ECTS (180 hours)

Lectures. Two 80 min lectures per week.

Practices. One 80 min practice session per week devoted mostly to discussing the homework assignments (see below).

Homework assignments. Weekly homework assignments (9 problem lists in total), published right after the practice session and due on the next practice session (in written form, preferably by email). The students may be asked to present their solutions in class. Each problem is worth 1 point, adding up to 45 points. Extra exercises (marked with a *) are to be submitted in writing for extra credit. The total number of points for homework assignments is capped at 50.

Final exam. The final written exam will take place in class. It will last 3 hours and consist of about 5 problems, worth 50 points in total.

Term paper (extra credit). A short (3-5 pages) paper explaining a chosen topic in algebraic geometry beyond what is taught in class, worth up to 20 points. Some possible topics will be suggested by the lecturer. The topic should first be discussed and approved by the lecturer.

The **final grade** will depend on the total score (out of 100 points).

Course Faculty

Piotr Achinger, Simons Professor in Mathematics at KSE
Academic Director of Graduate Programs in Mathematics
Associate Professor at IMPAN

Office hours: Tuesday, 10:00-12:00, KSE Dragon Capital Building, 3 Shpaka St., Front staircase, floor 5½

Piotr works in algebraic geometry and is broadly interested in the topological properties of algebraically defined geometric objects. He works at the Institute of Mathematics of the Polish Academy of Sciences

(IMPAN) in Warsaw (on leave during the 2025/26 academic year). Він розмовляє українською.

Piotr earned his PhD from the University of California, Berkeley, in 2015.

After holding postdoctoral positions at the Banach Center in Warsaw and the Institut des Hautes Études Scientifiques (IHES) in Paris, he became a researcher at the Institute of Mathematics of the Polish Academy of Sciences (IMPAN).

Piotr obtained his habilitation in 2022.

https://achinger.impan.pl/

Course Plan

Reading list

There is no required textbook. Highly relevant sources include the following books and online resources:

1. R. Hartshorne *Algebraic Geometry,* Springer Graduate Texts in Mathematics vol. 52, 1977
2. D. Mumford *The Red Book of Varieties and Schemes,* Springer Lecture Notes in Mathematics, vol. 1358
3. A. Grothendieck, J. Dieudonné *Éléments de géométrie algébrique (EGA), vol. I–IV,* Publications Mathématiques de l'IHÉS 1960–1967
4. R. Vakil *The Rising Sea: Foundations of Algebraic Geometry*, available at https://math.stanford.edu/~vakil/216blog/
5. The Stacks Project, https://stacks.math.columbia.edu/

Course plan (tentative)

1. Varieties over an algebraically closed field. Nullstellensatz and Noether normalization. Examples. Projective varieties. Rational maps.
2. Schemes and morphisms of schemes.
3. Quasi-coherent and coherent sheaves. Sheaf cohomology.
4. Divisors and line bundles. Ample line bundles and morphisms into projective space. Blowing up.
5. Separated and proper morphisms.
6. Differentials and smoothness. The canonical sheaf, Serre duality (without proof).
7. Curves. Riemann–Roch. Elliptic curves. Jacobians.
8. Intersection theory on surfaces. The Hodge index theorem.
9. The Weil conjectures for curves over finite fields.
10. A sketch of Hodge theory.